

## GLOBAL STABILITY OF TWO STRAINS MODEL WITH MUTATION, INFECTION AGE AND IMPULSIVE IMMUNITY

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**Abstract.** Since infection age and impulsive immunity are important factors of epidemic progression, we introduced the infection age and impulsive immunity into the model. In this paper, we analyze the dynamical behavior of our model and prove the existence of a strain-eradication periodic solution which is globally asymptotically stable if  $R_0^1 < 1$  and  $\int_0^T \beta_2 S^*(t) dt < (\mu + \gamma_2)T$ . If  $R_2 > 1$ ,  $R_0^1 < 1$ , the boundary solution is periodic. We simulate some dynamical behavior of this model. In our results, we have shown that for a short period of pulse  $T$  or a large pulse immunity rate  $p$  is the sufficient condition for the eradication of two strains.

**Keywords.** SIJR model; Infection age; Global attractor; Impulsive immunity; Persistence.

**AMS (MOS) subject classification:** 35K55; 92D25.

## 1 Introduction

Increase in disease virulence accelerates the transmission of infectious between hosts. Simultaneously it reduces the longevity of infection within hosts. Change in virulence can alter host-pathogen dynamics, significantly. The importance of subtypes in modeling the development and evolution of diseases is well recognized. Competitive exclusion and coexistence of strains in gonorrhoea and other sexually transmitted diseases are discussed in [1,2,13]. Some of those are super-infection<sup>[3,4,5]</sup>, coinfection<sup>[6]</sup>, cross-immunity<sup>[7]</sup>, density-dependent host mortality<sup>[8]</sup>. Mutation<sup>[9]</sup> is an important nature of the multi-strains. Mutation have impact on the complex nature of the strain epidemic model. Pathogen mutations that circumvent the protective effects of a patient's immune response are common in infections diseases such as measles, hepatitis B, HIV, West Nile virus, and influenza. In our paper, we consider the mutation of two strains epidemic model.

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