Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **18 (2011)** 149-161
 http://www.watam.org

LYAPUNOV INEQUALITIES FOR HALF-LINEAR DYNAMIC EQUATIONS ON TIME SCALES AND DISCONJUGACY

S. H. Saker

Department of Mathematics Skills, PY
King Saud University, Riyadh 11451, Saudi Arabia

Department of Math., Faculty of Science
Mansoura University, Mansoura 35516, Egypt
Corresponding author email:mathcoo@py.ksu.edu.sa

Abstract. In this paper we establish a time scale version of the Lyapunov inequality for half-linear dynamic equations on time scales which provides the lower bound for the distance between consecutive zeros of solutions. Also, we establish some sufficient conditions for disconjugacy and study the asymptotic behavior of the oscillatory solutions. These on the one hand generalizes and on the other hand furnish a handy tool for the study of qualitative as well as quantitative properties of solutions of dynamic equations on time scales.

Keywords. Dynamic equations; Time scale; Lyapunov inequality.

AMS (MOS) subject classification: 26D15; 26D20; 39A12.

1 Introduction

Consider the second-order half-linear dynamic equation

$$\left(r(t)\varphi(x^{\Delta})\right)^{\Delta}+p(t)\varphi(x^{\sigma}(t))=0, \tag{1}$$

on an arbitrary time scale $\mathbb{T}$, where $\varphi(u)=|u|^{\gamma-1}u$, $\gamma>0$ is a positive constant, r and p are real $rd-$continuous positive functions defined on $\mathbb{T}$ with $r(t)\neq 0$ and $\sigma(t)$ is the forward jump operator defined by $\sigma(t):=\inf\{s\in\mathbb{T}:s>t\}$. Our concern in this paper is to determine the lower bound for the distance between consecutive zeros of the solutions and establish some sufficient conditions for disconjuagacy as well as the study of the asymptotic behavior of the oscillatory solutions. Perhaps the best known existence results of this type for a special case of (1) (when $\mathbb{T}=\mathbb{R}$, $\gamma=1$ and $r(t)=1$) is due to Lyapunov. In [9] Lyapunov proved that if $y(t)$ is a solution of the differential equation

$$y^{''}(t)+p(t)y(t)=0, \tag{2}$$

with $y(a)=y(b)=0$ $(a<b)$ and $y(t)\neq 0$ for $t\in(a,b)$, then

$$\int_a^b |p(t)|\,dt>\frac{4}{b-a}. \tag{3}$$