Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **18 (2011)** 111-121
 http://www.watam.org

LIFE SPAN AND LARGE TIME BEHAVIOR FOR A DEGENERATE PARABOLIC EQUATION

Yuhuan Li[1] and Chunlai Mu[2]

[1]Department of Mathematics
Sichuan Normal University, Sichuan, Chengdu, P. R. China

[2]College of Mathematics and Statistics
Chongqing University, Chongqing, P. R. China

Corresponding author email:chunlaimu@yahoo.com.cn

Abstract. In this paper, we consider the positive solution of the Cauchy problem for the equation

$$u_t = u^p \Delta u + u^q, \quad p > 1, \quad q > 1,$$

and give an estimate of life span of non-global solutions of the Cauchy problem with initial value which behaves like $|x|^{-a}$ with $a > 0$ at infinite. Furthermore, the large time behavior of the solutions is also studied.

Keywords. Blow-up; Life span; Global existence; Large time behavior; Degenerate parabolic equation; Slowly decaying initial data.

AMS (MOS) subject classification: 35K55; 35K57.

1 Introduction

In his seminal paper, Fujita [5] considered the initial value problem

$$\begin{aligned} &u_t = \Delta u + u^p, \quad x \in R^N, \quad t > 0, \\ &u(x,0) = u_0(x), \quad x \in R^N, \end{aligned} \tag{1.1}$$

where $N \geq 1, p > 1$ and $u_0(x)$ is a bounded positive continuous function. In [5], it is shown that there is a critical exponent $p_1^* = 1 + \frac{2}{N}$ such that the solution $u(x,t)$ of (1.1) blows up in finite time for all $u_0(x)$ if $1 < p < p_1^*$ and there are global solutions and nonglobal solutions if $p > p_1^*$. In [19], Weissler has shown that p_1^* belongs to the blow-up case. A similar blow-up critical exponent for the following problem is also given when $p_m^* = m + \frac{2}{N}$ (see [6], [15], [16] and [18]).

$$\begin{aligned} &u_t = \Delta u^m + u^p, \quad x \in R^N, \quad t > 0, \\ &u(x,0) = u_0(x), \quad x \in R^N, \end{aligned} \tag{1.2}$$

where $p > 1, m > 1$ or $1 > m > \max\{0, 1 - \frac{2}{N}\}$ and $u_0(x)$ is a nonnegative bounded and continuous function. For more references on this topic, we