

AN ANALYTICAL INVARIANT MEASURE FOR RANDOM MAPS WITH POSITION DEPENDENT

S. Behnia¹, A. Akhshani^{1,2} and S. Ahadpour³

¹ Department of Physics, Faculty of Science, I.A.U., Orumieh Branch, Orumieh, Iran

² School of Physics, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia

³ Department of Physics, Mohaghegh Ardabili University, Ardabil, Iran
Corresponding author email: s.behnia@iaurmia.ac.ir

Abstract. The random map model is a deterministic dynamical system in a finite phase space with n points. The map that establishes the dynamics of the system is chosen randomly for every point. The essential idea of this paper is that given two dynamical systems that behave in a certain way, it is possible to combine these systems (by composition) into a new dynamical system which behaves in a completely different way than each of the constituent systems. We show that a dynamical system consisting of a chaotic map coupled to a map with noninvertible nonlinearity can generate random dynamics.

Keywords. Random map; Invariant measures; Perron-Frobenius operator; Ergodicity; Kolmogorov-Sinai(KS)entropy.

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1 Introduction

Maps are planar graphs embedded in the plane, and as such, they model the topology of many geometric arrangements in the plane and in spaces of low dimensional (e.g., 3-dimensional convex polyhedra). One-dimensional iterative maps represent a very useful tool to understand the physics of complex nonlinear systems. Ergodic theory of dynamical systems is concerned with the qualitative analysis of iterations of a single transformation. The ergodic nature of the chaotic dynamical system guarantees that it will eventually land in a neighborhood of the fixed point, however this may take a very long time to occur. Ulam and von Neuman [1] suggested the study of more general systems where, at each iteration, a transformation is selected randomly from a collection of transformations. Such dynamical systems have recently found application in the study of fractals [2] and in modeling interference effects in quantum mechanics [3]. A random map is discrete-time dynamical system in which one of a number of transformations is randomly selected and applied at each iteration of the process. Random map as a model was introduced by mathematicians half of a century ago [4], later on it was used to study statistical distribution [5]. Then it found a numerous application as a model