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# BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL EQUATIONS INVOLVING RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVE ON THE HALF-LINE

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**Abstract.** In this paper, we establish sufficient conditions for the existence of solutions for a class of boundary value problem for fractional differential equations involving the Riemann-Liouville fractional derivative on infinite intervals. This result is based on the nonlinear alternative of Leray-Schauder type combined with the diagonalization method.

**Keywords.** Boundary value problem; Differential equation; Riemann-Liouville fractional derivative; Fractional integral; Existence; Fixed point; Infinite intervals; Diagonalization process.

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## 1 Introduction

This paper deals with the existence of solutions for the boundary value problems (BVP for short) for fractional order differential equations of the form

$$D^{\alpha}y(t) = f(t, y(t)), \text{ for each } t \in J = [0, \infty), \quad 1 < \alpha \le 2, \tag{1}$$

$$y(0) = 0, \ y \text{ bounded on } [0, \infty), \tag{2}$$

where $D^{\alpha}$ is the Riemann-Liouville fractional derivative, $f : J \times \mathbb{R} \to \mathbb{R}$ is a given function.

Differential equations of fractional order have recently been proved to be valuable tools in the modeling of many phenomena in various fields of science and engineering. Indeed, we can find numerous applications in viscoelasticity, electrochemistry, control, porous media, electromagnetic, etc. (see [10, 11, 12, 15, 19, 20] and the references therein). There has been a