

CONVERGENCE OF SOLUTIONS OF A LINEAR IMPULSIVE DIFFERENTIAL EQUATIONS SYSTEM WITH MANY DELAYS

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Abstract. Sufficient conditions for convergence of solutions of a nonhomogeneous linear system of impulsive delay differential equations are obtained. Moreover, a limit formula is given.

Keywords. Impulsive delay differential equations system; Bounded solution; Convergence of the solution.

AMS subject classifications: 34K06; 34K45.

1 Introduction

In this paper, we consider the nonhomogeneous linear impulsive delay differential equations system

$$\begin{cases} x'(t) = A_0(t)x(t) + \sum_{l=1}^L A_l(t)x(t - \tau_l) + f(t), & t \geq t_0, t \neq \theta_i, \\ \Delta x(\theta_i) = B_0(i)x(\theta_i) + \sum_{k=1}^K B_k(i)x(\theta_{i-m_k}) + g(i), & i \in \mathbb{Z}^+ = \{0, 1, 2, \dots\}, \end{cases} \quad (1a)$$
$$\quad (1b)$$

where, $t_0 \geq 0$; $\tau_l > 0$ for $l = 1, 2, \dots, L$; $t_0 < \theta_0 < \theta_1 < \theta_2 < \dots < \theta_i < \dots$ and $\theta_i \rightarrow +\infty$ as $i \rightarrow +\infty$; $m_k \in \mathbb{Z}^+ - \{0\}$, $\theta_j \in [t_0 - \tau, t_0]$ for $j \in \{-m, 1-m, 2-m, \dots, -1\}$, $\tau = \max\{\tau_l : l = 1, 2, \dots, L\}$, $m = \max\{m_k : k = 1, 2, \dots, K\}$; $\Delta x(\theta_i) = x(\theta_i^+) - x(\theta_i^-)$, $x(\theta_i^+) = \lim_{t \rightarrow \theta_i^+} x(t)$, $x(\theta_i^-) = \lim_{t \rightarrow \theta_i^-} x(t)$. Moreover

we assume the following hypotheses:

(H₁) $A_l : [t_0, \infty) \rightarrow \mathbb{R}^{n \times n}$, $l = 0, 1, 2, \dots, L$, are continuous matrix functions,

(H₂) $f : [t_0, \infty) \rightarrow \mathbb{R}^n$ is a continuous vector function,

(H₃) $B_k(i) \in \mathbb{R}^{n \times n}$ for $k = 0, 1, 2, \dots, K$, $i \in \mathbb{Z}^+$,

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