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EXTREMAL SOLUTIONS FOR SECOND-ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS WITH NONLINEAR BOUNDARY CONDITIONS

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Abstract. This paper deals with the existence of extremal solutions for second-order three-point boundary value problems. We show the validity of the monotone iterative technique and improve some relevant results. As an application, an example is given to illustrate the results.

Keywords. Functional differential equation; Nonlinear boundary conditions; Three-point boundary problems; Lower and upper solutions; Monotone iterative method.

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1 Introduction

In recent years, many authors have paid attention to the research of boundary value problems for functional differential equations, which arise in a variety of different areas of Applied Mathematics and Physics [1, 3, 5-7, 10, 13]. For example, many problems in the theory of elastic stability can be handled by the method of multi-point problems [14]. Bridges of small size are often designed with two supported points, which leads to a standard two-point boundary condition and bridges of large size are sometimes contrived with multi-point supports, which corresponds to a multi-point boundary condition [15].

The method of upper and lower solutions coupled with the monotone iterative technique has been applied successfully to obtain existence and approximation of solutions for boundary value problem [4, 8, 9, 12]. Some attempts have been made to extend these techniques to study problems of functional differential equations. In [11], J. J. Nieto and R. Rodŕgue-López introduced a new concept of lower and upper solutions to study the following first order functional differential equation:

$$\begin{cases} u'(t) = g(t, u(t), u(\theta(t))), \ t \in [0, T], \\ u(0) = u(T). \end{cases}$$