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INTEGER VALUED FIXED POINT INDEX FOR COMPACT ACYCLIC MAPS

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Abstract. An index theory is presented for compositions of **Z**-acyclic w.r.t. the Čech cohomology with integer coefficients, compact maps and several new fixed point theorems are given for such maps.

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1 Introduction

In Section 2 we present the fixed point index for compositions of **Z**-acyclic w.r.t. the Čech cohomology with integer coefficients, compact (or more generally compact absorbing contractive) multivalued maps. In addition we introduce a new class of maps which we call the $\mathcal{SK}$ maps. In Section 3 we provide an alternative approach to establishing fixed point theory using projective limits. These results improve those in the literature; see [1-9] and the references therein.

Let X be a metric space, U an open subset of X and $F:\overline{U}\to K(X)$ (here $K(X)$ denotes the family of nonempty compact subsets of X) an upper semicontinuous map. We say $F\in\mathcal{K}(\overline{U},X)$ if

(a). there is a natural number k such that $F=F_{k-1}\,.....\,F_1\,:\overline{U}\to X$ where $F_i:X_i\to X_{i+1}$, $i=1,...,k-1$, $X_1=\overline{U}$ and $X_k=X$;

(b). the maps F_i, $i=1,....,k-1$, are **Z**-acyclic w.r.t. the Čech cohomology with integer coefficients.

In this case we say F is determined by the decomposition $(F_1,...,F_{k-1})$.

With X, F and U as above we have a homomorphism $F^\star=F_1^\star......F_{k-1}^\star:H^\star(X;\mathbf{Z})\to H^\star(\overline{U};\mathbf{Z})$ where $F_i^\star:H^\star(X_{i+1};\mathbf{Z})\to H^\star(X_i;\mathbf{Z})$ is defined as follows: consider the representation $F_i=q_i\,p_i^{-1}$ with the projections $p_i:\Gamma(F_i)\to X_i$ (here $\Gamma(F_i)$ is the graph of F_i) and $q_i:\Gamma(F_i)\to X_{i+1}$ and the Vietoris theorem implies that $p_i^\star:H^\star(X_i;\mathbf{Z})\to H^\star(\Gamma(F_i);\mathbf{Z})$ is an isomorphism, so $F_i^\star=(p_i^\star)^{-1}q_i^\star:H^\star(X_{i+1})\to H^\star(X_i)$.