Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **18 (2011)** 17-28

http://www.watam.org

APPROXIMATE GREATEST DESCENT METHOD AND QUASI-NEWTON MATRICES IN OPTIMIZATION

B. S. Goh[1], Zheng Peng[2], C.S. Lee[3], Junfeng Yang[4], Min Kong[4]

[1]Institute of Mathematical Sciences,
University of Malaya, 50603 Kuala Lumpur, Malaysia.

[2]College of Mathematics and Computer Science,
Fuzhou University, 350108, Fuzhou, P. R. China.

[3] Former Associate Professor, Mathematics and Mathematics Education,
NIE, Nanyang Technological University, Singapore.

[4] Mathematics Department, Nanjing University, Nanjing, Jiangsu 210093, China.

Abstract. The long-term optimal trajectory to compute a minimum point consists of a sequence of greatest descent steps followed by the Newton step in the last iteration. The greatest descent direction can be approximated by a Levenberg-Marquardt like formula. There is a simple way to prescribe the relative steplengths so that the approximate greatest descent (AGD) direction merges into the Newton direction near a minimum point. This enables fast local convergence of the AGD method near a minimum point. Here we examine the use of the B_k matrices, defined by a quasi-Newton update formula, as a way to model and approximate the Hessian matrix of a nonlinear function. These B_k matrices are used in the AGD iteration rather than the Newton iteration. Used in this manner numerical errors in the B_k matrices can be tolerated when the point is at a large distance from the minimum point. Furthermore, B_k is not required to be positive definite or nonsingular. Instead we require a weaker condition, namely the function is monotonic decreasing. This can always be achieved by using a small steplength. Computational errors can make the B_k matrix singular and not positive definite. From numerical experiments the main advantage of using the B_k matrix rather than the Hessian matrix in the AGD method is that it is faster when the number of variables is large.

Keywords. Unconstrained optimization; greatest descent direction; Newton method; quasi-Newton method; convergence.

AMS (MOS) subject classification: 90C30, 97N60, 65K05.

1 Introduction

Goh (2009) has shown that the structure of a long term optimal trajectory to compute the minimum point of an unconstrained function consists of a sequence of greatest descent steps and the Newton method as the last step. The

[1]B.S. Goh, Corresponding author, Professor, Institute of Mathematical Sciences, University of Malaya, 50603 Kuala Lumpur, Malaysia. E-mail: goh2optimum@gmail.com