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AN EQUIVALENT PROBLEM APPROACH TO ABSOLUTE EXTREMA FOR CALCULUS OF VARIATIONS PROBLEMS WITH DIFFERENTIAL CONSTRAINTS

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Abstract. We consider problems in the calculus of variations of the form

$$\text{minimize } \mathcal{J}(x(\cdot)) = \int_a^b L(t, x(t), \dot{x}(t))\, dt$$

over the class of all absolutely continuous functions satisfying the constraint

$$g(t, x(t), \dot{x}(t)) = 0, \quad \text{for almost all} \quad t \in [a, b],$$

and the end conditions

$$x(a) = x_a \quad \text{and} \quad x(b) = x_b,$$

where $(L, g)(\cdot,\cdot,\cdot) : [a, b] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R} \times \mathbb{R}^m$ are continuous functions with $g(t, x, p) \geq 0$ (componentwise) for all $(t, x, p) \in [a, b] \times \mathbb{R}^n \times \mathbb{R}^n$ and $m < n$. Our approach to solving such problems is to combine a penalization method with Leitmann's direct sufficiency method. More specifically we consider the family of unconstrained problems $(\mathcal{P}_\lambda)$ of minimizing

$$\mathcal{J}_\lambda(x(\cdot)) \doteq \int_a^b L(t, x(t), \dot{x}(t)) + \lambda^{\mathsf{T}} g(t, x(t), \dot{x}(t))\, dt \tag{1}$$

over all piecewise smooth functions $x(\cdot)$ satisfying (3). The parameter $\lambda \in \mathbb{R}^m$ is assumed to be a constant vector whose components are all the same positive constant. Our goal is to apply Leitmann's direct method to the penalized problem with sufficiently large λ. An example will be presented to illustrate our technique.

Keywords.calculus of variations, differential side conditions, sufficient conditions.

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1 Introduction

Recently, the authors have been investigating a direct sufficiency method for solving free problems in the calculus of variations. This approach involves

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