

EXISTENCE RESULTS FOR IMPULSIVE FUNCTIONAL DIFFERENTIAL INCLUSIONS

L. Górniewicz,¹ S.K. Ntouyas² and D. O'Regan³

¹Faculty of Mathematic and Informatic Science, Nicholas Copernicus University
Chopina 12/18, 87-100 Torun, Poland
e-mail: gorn@mat.uni.torun.pl

²Department of Mathematics
University of Ioannina, 451 10 Ioannina, Greece
e-mail: sntouyas@cc.uoi.gr

³Department of Mathematics
National University of Ireland, Galway, Ireland
e-mail: donal.oregan@unigalway.ie

Abstract. In this paper we prove existence results for first and second order impulsive functional and neutral functional differential inclusions in Banach spaces.

Keywords. Impulsive functional differential inclusions, fixed point.

AMS (MOS) subject classification: 34A60, 34K05.

1 Introduction

In this paper we prove existence results for initial value problems for first and second order impulsive functional differential inclusions. First, in Section 3 we study first order impulsive functional differential inclusions of the form

$$y'(t) \in F(t, y_t), \quad \text{a.e. } t \in J := [0, T], \quad t \neq t_k, \quad k = 1, \dots, m, \quad (1)$$

$$\Delta y|_{t=t_k} = I_k(y(t_k^-)), \quad k = 1, \dots, m, \quad (2)$$

$$y(t) = \phi(t), \quad t \in [-r, 0], \quad (3)$$

where $F : J \times \mathcal{D} \rightarrow \mathcal{P}(E)$ is a multivalued map, $\mathcal{D} = \{\psi : [-r, 0] \rightarrow E \mid \psi \text{ is continuous everywhere except for a finite number of points } s \text{ at which } \psi(s) \text{ and the right limit } \psi(s^+) \text{ exist and } \psi(s^-) = \psi(s)\}$, $\phi \in \mathcal{D}$, $(0 < r < \infty)$, $0 = t_0 < t_1 < \dots < t_m < t_{m+1} = T$, $I_k \in C(E, E)$ ($k = 1, 2, \dots, m$), E a real separable Banach space with norm $|\cdot|$ and $\mathcal{P}(E)$ is the family of all subsets of E .

For any continuous function y defined on the interval $[-r, T] \setminus \{t_1, \dots, t_m\}$ and any $t \in J$, we denote by y_t the element of \mathcal{D} defined by

$$y_t(\theta) = y(t + \theta), \quad \theta \in [-r, 0].$$