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ON SOLUTION SETS FOR FIRST ORDER IMPULSIVE NEUTRAL FUNCTIONAL DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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Abstract. In this paper, we first present an impulsive version of Filippov's Theorem for first-order neutral functional differential inclusions of the form,

$$\begin{array}{rcll} \frac{d}{dt}[y(t)-g(t,y_t)] & \in & F(t,y_t), & \text{a.e. } t\in J\backslash\{t_1,\dots,t_m\}, \\ y(t_k^+)-y(t_k^-) & = & I_k(y(t_k^-)), & k=1,\dots,m, \\ y(t) & = & \phi(t), & t\in[-r,0], \end{array}$$

where $J=[0,b]$, F is a set-valued map and g is a single-valued function. The functions I_k characterize the jump of the solutions at impulse points t_k $(k=1,\dots,m)$. Then the convexified problem is considered and a Filippov-Ważewski result is proved. After several existence results, the topological structure of solution sets is also investigated. Some results from topological fixed point theory together with notions of measure of noncompactness are used. Finally, some geometric properties of solution sets are obtained. Applications to a problem from control theory are provided.

Keywords. Impulsive functional differential inclusions; Neutral; Filippov's theorem; Relaxation; Solution set; Compactness; AR; R_δ; Contractibility; Acyclicity; Control.

AMS (MOS) subject classification: 34A37, 34A60, 34K30, 34K45.

1 Introduction

It is well-known that systems with after effect, with time lag or with delay, are of great theoretical interest and form an important class with regard to their applications. This class of systems can be described by functional differential equations and inclusions, which are also called differential equations and inclusions with deviating argument. Among functional differential equations, one may distinguish some special classes of equations, retarded functional differential equations, advanced functional differential equations and neutral functional equations and inclusions. In particular, retarded functional differential equations and inclusions describe those systems or processes whose rate of change of state is determined by their past