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POSITIVE SOLUTIONS TO BOUNDARY VALUE PROBLEMS FOR IMPULSIVE SECOND–ORDER DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we discuss four–point boundary value problems for impulsive second–order differential equations. We apply the Krasnoselskii's fixed point theorem to obtain sufficient conditions under which the impulsive second–order differential equations have positive solutions. An example is added to illustrate theoretical results.

Keywords. Impulsive differential equations; Four–point boundary value problems; Positive solutions.

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1 Introduction

For $J = [0, 1]$, let $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$. Put $J' = (0, 1) \setminus \{t_1, t_2, \cdots, t_m\}$. Put $\mathbb{R}_+ = [0, \infty)$ and $J_k = (t_k, t_{k+1}],\ k = 0, 1, \cdots, m-1,\ J_m = (t_m, t_{m+1})$.

Let us consider second–order impulsive differential equations of type

$$\begin{cases} x''(t) + \lambda h(t) f(x(t)) = 0, \quad t \in J', \\ \Delta x'(t_k) = Q_k(x(t_k)), \quad k = 1, 2, \cdots, m, \\ x(0) = \gamma x(\xi), \quad x(1) = \beta x(\eta), \quad \xi, \eta \in (0, 1), \end{cases} \tag{1}$$

where as usually $\Delta x'(t_k) = x'(t_k^+) - x'(t_k^-)$; $x'(t_k^+)$ and $x'(t_k^-)$ denote the right and left limits of x' at t_k, respectively. Here, $\lambda > 0$ is a parameter and $\gamma, \beta > 0$. Note that if $\xi = \eta$, then (1) reduces to a three–point problem.

We assume that:

A_1 : $f \in C(\mathbb{R}_+, \mathbb{R}_+)$, and there exist nonnegative constants in the extended reals, f_0, f_∞, such that

$$f_0 = \lim_{u \to 0^+} \frac{f(u)}{u}, \quad f_\infty = \lim_{u \to \infty} \frac{f(u)}{u};$$

$h \in C(J, \mathbb{R}_+)$ and h does not vanish identically on any subinterval;