

SOME COMMON FIXED POINT THEOREMS FOR WEAKLY COMMUTING MAPS IN \mathcal{L} -FUZZY METRIC SPACES

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Abstract. In this paper we prove two common fixed point theorems in generalized fuzzy and intuitionistic fuzzy metric spaces i.e., \mathcal{L} -fuzzy metric space.

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1 Introduction and Preliminaries

The notion of fuzzy sets was introduced by Zadeh [22]. Various concepts of fuzzy metric spaces were considered in [6, 7, 12, 13]. Many authors have studied fixed point theory in fuzzy metric spaces; see for example [2, 3, 9, 10, 11, 15, 16, 21]. In the sequel, we shall adopt the usual terminology, notation and conventions of \mathcal{L} -fuzzy metric spaces introduced by Saadati et al. [18] which are a generalization of fuzzy metric spaces [8] and intuitionistic fuzzy metric spaces [17, 19].

Definition 1.1 ([9]) Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice, and U a non-empty set called a universe. An \mathcal{L} -fuzzy set \mathcal{A} on U is defined as a mapping $\mathcal{A} : U \rightarrow L$. For each u in U , $\mathcal{A}(u)$ represents the degree (in L) to which u satisfies \mathcal{A} .

Lemma 1.2 ([4, 5]) Consider the set L^* and the operation \leq_{L^*} defined by:

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0, 1]^2 \text{ and } x_1 + x_2 \leq 1\},$$

$(x_1, x_2) \leq_{L^*} (y_1, y_2) \iff x_1 \leq y_1 \text{ and } x_2 \geq y_2$, for every $(x_1, x_2), (y_1, y_2) \in L^*$. Then (L^*, \leq_{L^*}) is a complete lattice.

Classically, a triangular norm T on $([0, 1], \leq)$ is defined as an increasing, commutative, associative mapping $T : [0, 1]^2 \rightarrow [0, 1]$ satisfying $T(1, x) = x$,

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