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NONLINEAR ANALYSIS OF RING OSCILLATOR AND CROSS-COUPLED OSCILLATOR CIRCUITS

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Abstract. Hassan Khalil's research results and beautifully written textbook on nonlinear systems have influenced generations of researchers, including the authors of this paper. Using nonlinear systems techniques, this paper analyzes ring oscillator and cross-coupled oscillator circuits, which are essential building blocks in digital systems. The paper first investigates local and global stability properties of an n-stage ring oscillator by making use of its cyclic structure. It next studies global stability properties of a class of cross-coupled oscillators which admit the representation of a dynamic system in feedback with a static nonlinearity, and presents sufficient conditions for almost global convergence of the solutions to a limit cycle when the feedback gain is in the vicinity of a bifurcation point. The result are also extended to the synchronization of interconnected identical oscillator circuits.

Keywords. Ring oscillators, cross-couple oscillators, cyclic structure, stability, synchronization.

1 Introduction

Due to their integrated nature, voltage-controlled oscillators (VCOs) are widely used in commercial applications. Ring oscillators and cross-coupled oscillators are two important types of VCOs in many electronic systems. Ring oscillators are used in applications such as clock recovery circuits [1] and disk-drive read channels [2], while cross-coupled oscillators can be employed in function generators, frequency synthesizers, etc. Some of the early stability studies were based on experimental results and linear analysis tools. Reference [3] used the Nyquist stability criterion to analyze the stability of a linearized oscillator. Reference [4] applied linear theory to the analysis of oscillation-frequency and oscillation-amplitude stability of nonlinear feedback oscillators. The global characteristics of oscillators, however, can be understood only with nonlinear systems techniques.

In addition to nonlinear analysis of individual oscillators, synchronization in systems of identical or nearly identical coupled oscillators has long been a topic of interest. Networks of synchronized oscillator circuits are being increasingly employed in digital communications to account for the frequency drift of individual oscillators and to accommodate more than one digital standard. A key problem in the