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ADAPTIVE CONTROL OF AN ANTI-STABLE WAVE PDE

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Abstract. Adaptive control of PDEs is a problem of nonlinear dynamic feedback design for an infinite-dimensional system. The problem is nonlinear even when the PDE is linear. Past papers on adaptive control of unstable PDEs with unmatched parametric uncertainties have considered only parabolic PDEs and first-order hyperbolic PDEs. In this paper we introduce several tools for approaching adaptive control problems of second-order-in-time PDEs. We present these tools through a benchmark example of an unstable wave equation with an unmatched (non-collocated) anti-damping term, which serves both as a source of instability and of parametric uncertainty. This plant has infinitely many eigenvalues arbitrarily far to the right of the imaginary axis and they reside on a vertical line whose position is completely unknown. The key effort in the design is to avoid the appearance of the second time derivative of the parameter estimate in the error system.

Keywords: Adaptive control, distributed parameter systems, backstepping, boundary control.

1 Introduction

Background. Adaptive control of infinite-dimensional systems is a challenging topic to which several researchers have contributed over the last two decades [4, 5, 8, 12, 13, 14, 16, 17, 18, 19, 20, 21, 27, 28, 34, 36]. The results have either allowed plant instability but required distributed actuation, or allowed boundary control but required that the plant be at least neutrally stable.

Recently we introduced several designs for non-adaptive [30] and adaptive [26, 31, 32] boundary control of unstable parabolic PDEs. Subsequently, in [6] we also tackled systems with unknown input delay, i.e., an important class of infinite-dimensional systems with first-order hyperbolic PDE dynamics. The remaining major class of PDEs for which adaptive boundary control results have not been developed yet, at least not in the case where the plant is unstable, are second-order hyperbolic PDEs, namely wave equations. Wave (and beam) equations have been tackled in [5, 8, 12, 18, 20, 21, 28, 34, 36], however, not *unstable* ones.

Contributions of the Paper. In this paper we present the first adaptive control design for an unstable wave equation controlled from a boundary, and where the source of instability is not collocated (matched) with control. We focus on the (notationally) simplest problem, but a problem that, among all basic wave equation