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AVERAGING FOR A CLASS OF HYBRID SYSTEMS

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Abstract. Averaging theory for ordinary differential equations is extended to a class of hybrid systems. Rapid time variations in the flow map of a hybrid system generate solutions that are also solutions of a slightly perturbed time-invariant average hybrid system. Results relating solutions of the time-varying system to solutions of the average system ensue. In the absence of finite escape times for the average system, on compact time domains each solution of the time-varying system is close to a solution of the average system. If the average system is asymptotically stable, the time-varying system exhibits semi-global, practical asymptotic stability. These results rely on mild regularity properties for the average system. In particular, the average system is not required to exhibit unique solutions. Both periodic and non-periodic flow maps are considered. The results are partially motivated by the desire to justify a pulse-width modulated implementation of hybrid feedback control for nonlinear systems.

Keywords. Averaging, nonlinear systems, stability, pulse width modulated control.

1 Introduction

Averaging theory exploits a time-scale separation between the time variations of the state of a dynamical system and the time variations of the derivative of that state. The theory justifies the use of a simplified - in particular, averaged - system to approximate the original system. A concise overview of averaging theory is provided by Hassan Khalil in the Control Handbook [13]. This article, which extends averaging theory to hybrid systems, is dedicated to Professor Khalil, who has exploited multiple time-scale phenomena cleverly throughout his illustrious research career.

Averaging theory for ordinary differential equations has a rich history, dating to back to the work of Krylov and Bogoliubov [15], and has been used extensively in engineering applications, including adaptive control [24], vibrational control [19], and to justify the implementation of feedback through pulse-width modulation [16], [28]. Books that cover averaging theory for continuous-time systems include [11], [14] and [23].