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PERSISTENCE AND TIME-VARYING NONLINEAR CONTROLLERS FOR A CHEMOSTAT WITH MANY SPECIES

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Abstract. This paper is devoted to the persistence issue for chemostat models with an arbitrary number of species competing for a single limiting substrate. On the one hand, fundamental limitations of nonlinear feedback control exist for the persistence in some chemostats. That is, there exist some bounded periodic trajectories for which there is no feedback control law guaranteeing trajectory-stabilization. On the other hand, for other families of growth rates, it is shown that a dilution rate and input substrate time-varying nonlinear controllers can be designed so that a positive trajectory of the chemostat model becomes globally asymptotically stable. In this case, the designed control laws ensure persistence of all the species. A local version of this result is given in the situation where only the substrate concentration is available for feedback design.

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1 Introduction

The basic model for a chemostat with N competing species and one limiting substrate is

$$\left\{\begin{array}{rcl} \dot{s} & = & D[S_{\rm in} - s] - \displaystyle\sum_{i=1}^{N} \mu_i(s) x_i, \\ \dot{x}_i & = & [\mu_i(s) - D] x_i \ , \ \forall i = 1, ..., N \ , \end{array}\right. \tag{1}$$

that evolves on $\mathcal{X} := (0, \infty)^{N+1}$. The interested reader is referred to [17], [5], [6] for the details on this celebrated model. It is worth noting that the case of different removal rates is studied in [11] while the authors of [18] focus on models of the chemostat with delays. The variable s denotes the substrate concentration and each variable x_i denotes the concentration of the ith species, the species being characterized by its growth function μ_i (which, in many cases, are increasing functions). In the system (1), the yield