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CHARACTERIZATION THEOREM FOR BEST LINEAR SPLINE APPROXIMATION WITH FREE KNOTS

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Abstract. A necessary condition for a best Chebyshev approximation by piecewise linear functions is derived using quasidifferential calculus. We first discover some properties of the knots joining the linear functions. Then we use these properties to obtain the optimality condition. This condition is stronger than existing results. We present an example of linear spline approximation where the existing optimality conditions are satisfied, but not the proposed one, which shows that it is not optimal.

Keywords. Stationarity conditions, quasidifferentials, Chebyshev approximation, linear splines.

AMS (MOS) subject classification: 49J52, 90C26, 41A50, 41A15.

1 Introduction

We consider the problem of best data approximation in an interval by piecewise linear functions, called also linear splines. The spline functions are nondifferentiable at the points where the linear functions meet (knots). Our motivation is to apply nonsmooth optimization tools to improve the existing results in the case of splines with free knots. This problem has been identified as an important open problem [4], as it is acknowledged that the existing tools are not adapted to this problem, due to its nonconvex and nonsmooth nature.

When the knots are known (fixed knots) the problem is convex. Necessary and sufficient optimality conditions for fixed knots polynomial spline approximation are presented in [10, 12, 14]. These conditions are generalisations of the well-known Chebyshev optimality condition for approximation by polynomials [5].

In the case of free knots polynomial (including linear) spline approximation the corresponding optimisation problems are nonconvex and therefore necessary and sufficient conditions often do not coincide. The search for such conditions has been studied by several researchers. [10] gives a good review of existing results. In the current paper we restrict ourselves to linear splines.