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GLOBAL OPTIMALITY CONDITIONS FOR MIXED INTEGER WEAKLY CONCAVE PROGRAMMING PROBLEMS

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Abstract. In this paper, some necessary and some sufficient global optimality conditions for a class of mixed integer programming problems whose objective functions are the difference of quadratic functions and convex functions are established. The numerical examples are also presented to show the significance of the global optimality conditions for this class of programming problems.

Keywords. Global optimality conditions, mixed integer programming problems, weakly concave programming problems.

AMS (MOS) subject classification: 41A65, 41A29, 90C30.

1 Introduction

Consider the following mixed integer programming problem:

$$
\begin{aligned}
(MIP) \qquad & \min \quad f(x) = \frac{1}{2}x^T Ax + d^T x - g(x) & (1.1)\\
& \text{s.t.} \quad x_i \in [u_i, v_i], i \in L, & \\
& \qquad\;\; x_j \in \{p_j, p_j + 1, \ldots, q_j\}, j \in M &
\end{aligned}
$$

where A is a $n \times n$ symmetric matrix, $d \in \mathbb{R}^n$, $g(x) : \mathbb{R}^n \to \mathbb{R}$ is a twice continuously differentiable convex function on $\mathbb{R}^n$, $L, M \subset \{1, \ldots, n\}$ with $L \cap M = \emptyset$ and $L \cup M = \{1, \ldots, n\}$, $u_i, v_i \in \mathbb{R}$ with $u_i < v_i, \forall\ i \in L$, and p_j, q_j are integers with $p_j < q_j, \forall j \in M$. We call problem (MIP) as a mixed integer weakly concave programming problem. Throughout the paper, we let

$$\mathcal{F} := \{x = (x_1, \ldots, x_n)^T \mid x_i \in [u_i, v_i], i \in L; x_j \in \{p_j, p_j{+}1, \ldots, q_j\}, j \in M\}.$$