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ABSTRACT CONVEXITY OF EXTENDED REAL VALUED INCREASING AND POSITIVELY HOMOGENEOUS FUNCTIONS

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Abstract. In this paper, we examine abstract convexity of extended real valued increasing and positively homogeneous functions defined on ordered topological vector spaces. We also study many other associated properties like the description of support set, polar function and subdifferential. Finally, we give a characterization for upward sets in a very general setting.

Keywords. Monotonic analysis, IPH function, upward set, co-radiant set, abstract convexity.

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1 Introduction

The theory of IPH functions can be applied to study of some NTU games arising in mathematical economics [1, 14] and to analyze of topical functions, which are used in the analysis discrete event system [4, 5, 6]. Some results about monotonic analysis on the space $\mathbb{R}^n$ with respect to the coordinate-wise order relation can be found in [8, 9, 12, 13].
An extension of this theory which is defined on a closed convex cone K in a topological vector space has been shown in [2, 3]. There are two main results of this theory. First, each IPH function p defined on K can be represented as the Minkowski gauge of a normal closed along rays subset of K, namely, $U = \{x \in K \ : \ p(x) \leq 1\}$, and vice versa. The second result is based on ideas of abstract convexity. On the other hand, each IPH function is supremally generated by the set of elementary functions which are IPH functions. A suitable extension for non-negative IPH functions on the whole of the space has been shown in [10]. On the other hand (see [[10], Theorem 3.2]), the function $p : X \to [0, +\infty]$ is an IPH function if and only if p is abstract convex with respect to a certain class of IPH functions. In this paper, we are going to extend the above results for extended real valued IPH functions defined on a topological vector space.