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OVERTAKING OPTIMAL SOLUTIONS FOR A CLASS OF INFINITE HORIZON DISCRETE-TIME OPTIMAL CONTROL PROBLEMS

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Abstract. In this paper we establish the existence of overtaking optimal solutions for a large class of infinite horizon discrete-time optimal control problems. This class contains optimal control problems arising in economic dynamics which describe a model proposed by Robinson, Solow and Srinivasan with nonconcave utility functions representing the preferences of the planner.

Keywords. Compact set, constraint mapping, infinite horizon problem, program, Robinson-Solow-Srinivasan model.

AMS (MOS) subject classification: 49J99.

1 Introduction

The study of the existence and the structure of solutions of optimal control problems defined on infinite intervals and on sufficiently large intervals has recently been a rapidly growing area of research. See, for example, [3, 6, 16, 19-21, 25-27, 28, 29, 37] and the references mentioned therein. These problems arise in engineering [1, 17], in models of economic growth [2-4, 8-15, 20-23, 25, 30-33, 35-37], in infinite discrete models of solid-state physics related to dislocations in one-dimensional crystals [5, 34] and in the theory of thermodynamical equilibrium for materials [7, 18, 24]. In this paper we study a large class of infinite horizon discrete-time optimal control problems. This class contains optimal control problems arising in economic dynamics which describe a model proposed by Robinson, Solow and Srinivasan with nonconcave utility functions representing the preferences of the planner. This model was recently studied in [13-15, 35, 36].

We begin with some preliminary notation. Let R (R_+) be the set of real (non-negative) numbers and let R^n be a finite-dimensional Euclidean space with non-negative orthant $R^n_+ = \{x \in R^n : \ x_i \geq 0, \ i = 1, \dots, n\}$. For any $x, y \in R^n$, let the inner product $xy = \sum_{i=1}^n x_i y_i$, and $x >> y$, $x > y$, $x \geq y$ have their usual meaning. Let $e(i)$, $i = 1, \dots, n$, be the ith unit vector in R^n, and e be an element of R^n_+ all of whose coordinates are unity. For any $x \in R^n$, let $||x||_2$ denote the Euclidean norm of x.