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PERIODIC AND HOMOCLINIC ORBITS FOR A CLASS OF IMPULSIVE DIFFERENTIAL EQUATIONS

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Abstract. In the present paper, we develop a variational approach to study the existence of periodic and homoclinic orbits for a class of second order impulsive differential equations. First, for any positive integer l, we prove the existence of at least one non-zero $lT-$periodic orbits for the impulsive system via Mountain Pass Lemma. Then using $2lT-$periodic orbits as approximations of the homoclinic orbits, we prove the existence of at least one non-zero homoclinic orbit of the impulsive system.

Keywords. Impulsive differential equation; periodic orbit; homoclinic orbit; mountain pass lemma; critical point theory.

AMS (MOS) subject classification: 34A37; 34B37; 34C25; 34C37.

1 Introduction

Impulsive differential equations arise from real world and are used to describe the dynamics of processes which possess sudden, discontinuous jumps. Such processes naturally occur in control theory, optimization theory, population dynamics and biology, and some physics and mechanics problems. Due to its significance, a lot of effort has been made in the theory of impulsive differential equations. For general aspects of impulsive differential equations, monographs [4, 8] are recommended.

For second order impulsive differential equations, one usually considers the impulses happening both in the position q and in the velocity $\dot{q}$. However, in the processes describing the motion of spacecrafts and satellites, the impulses only happen in the velocity with no jumps in the position [1, 5]. In this paper we will study such a system. More precisely, in this paper we will investigate the existence of periodic and homoclinic orbits for the following second order impulsive differential equations,

$$\ddot{q} + V_q(t,q) = f(t), \qquad \text{for } t \in (s_{k-1}, s_k), \tag{1.1}$$

$$\Delta\dot{q}(s_k) = g_k(q(s_k)), \tag{1.2}$$

where $k \in \mathbb{Z}$, $q(t) \in \mathbb{R}^n$, $\Delta\dot{q}(s_k)) = \dot{q}(s_k^+) - \dot{q}(s_k^-)$ with $\dot{q}(s_k^\pm) = \lim_{t\to s_k\pm} \dot{q}(t)$, $V_q(t,q) = \operatorname{grad}_q V(t,q)$, $f \in C(\mathbb{R},\mathbb{R}^n)$, $g_k(q) = \operatorname{grad}_q G_k(q)$ with $G_k \in$