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OSCILLATORY BEHAVIOR OF THIRD ORDER NONLINEAR DIFFERENCE EQUATIONS WITH DELAYED ARGUMENT

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Abstract. By means of the Riccati transformation technique, we will establish some new oscillation criteria for certain class of third order nonlinear difference equations with delayed argument. Our results extend and improve some well known ones in the literature. Some examples are worked out to demonstrate the main results.

Keywords. Oscillation; Riccati transformation; Third order delay difference equation.

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1 Introduction

Consider the nonlinear delay difference equation

$$\Delta(c_n\Delta(d_n\Delta x_n)^{\gamma}) + q_n x_{n-\sigma}^{\gamma} = 0, \quad n \geq n_0, \tag{1}$$

where n_0 is a nonnegative integer, Δ denotes the forward difference operator $\Delta x_n = x_{n+1} - x_n$ for any sequence $\{x_n\}$ of real numbers, $\gamma \geq 1$ is quotient of odd positive integers, σ is a nonnegative integer and the real sequences $\{c_n\}_{n=n_0}^{\infty}$, $\{d_n\}_{n=n_0}^{\infty}$ and $\{q_n\}_{n=n_0}^{\infty}$ satisfy the following conditions:

(h1) $\{c_n\}_{n=n_0}^{\infty}$ and $\{d_n\}_{n=n_0}^{\infty}$ are positive sequences of real numbers such that $\sum_{n=n_0}^{\infty}\left(\frac{1}{c_n}\right) = \sum_{n=n_0}^{\infty}\left(\frac{1}{d_n}\right) = \infty$;

(h2) $q_n \geq 0$ and $\{q_n\}_{n=n_0}^{\infty}$ has a positive subsequence.

By a solution of (1), we mean a nontrivial real sequence $\{x_n\}$ that is defined for $n \geq n_0 - \sigma$ and satisfies equation (1) for $n \geq n_0$. One can easily see that if

$$x_n = \phi_n, \;\; \text{for} \quad n = n_0 - \sigma,\; n_0 - \sigma + 1, \ldots,\; n_0 - 1 \tag{2}$$