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EXISTENCE AND REGULARITY OF SOLUTIONS TO NON-AUTONOMOUS FUNCTIONAL EVOLUTION EQUATIONS WITH INFINITE DELAY [1]

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Abstract. In this paper, by using Sadovskii fixed point theorem, we study the existence and regularity of mild solutions for a class of abstract neutral functional evolution equations with infinite delay, where the linear part is non-autonomous and generates a linear evolution system. The fraction power theory and $\alpha-$norm is used to discuss the problem so that the obtained results can be applied the equations which involve spatial derivatives. An example is presented to show the applications of the obtained results.

Keywords. Neutral functional evolution equation, linear evolution operator, fractional power operator, infinite delay.

AMS (MOS) subject classification: 34K25, 34K30, 34G20.

1 Introduction

In this paper, we investigate the existence and regularity of mild solutions for the following abstract neutral functional evolution equation with infinite delay:

$$\begin{cases} \frac{d}{dt}[x(t) + F(t, x_t)] + A(t)x(t) = G(t, x_t), & 0 \le t \le a, \\ x_0 = \phi \in \mathscr{B}_\alpha. \end{cases} \tag{1.1}$$

where $x(\cdot)$ takes values in a subspace of Banach space X, the family $\{A(t) : 0 \le t \le a\}$ of unbounded linear operators generates a linear evolution operator $U(t, s)$, $0 \le s \le t \le a$, and $F,\ G : [0, a] \times \mathscr{B}_\alpha \to \mathscr{B}$ are appropriate functions, where $\mathscr{B}_\alpha \subset \mathscr{B}$, and $\mathscr{B}$ is the phase space to be specified later.

Since a lot of practical functional differential models can be studied by rewritten to this kind of abstract equations, in these years there has been an increasing interest in the study of semilinear evolution equations of form (1.1), such as existence and asymptotic behavior of solutions (mild solutions, strong solutions and classical solutions), and existence of (almost) periodic solutions, etc. Here we only mention the work of Travis and Webb[20], Fitzgibbon[6], Rankin III[17] for the case of finite delay, and Henriquez[10], Liu[14], Diagana and Hernández[3] for the case of

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