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HOPF BIFURCATION FOR A CLASS OF COMPETITION REACTION-DIFFUSION SYSTEM WITH DELAY

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Abstract. In this work we have studied a class of competition reaction-diffusion system with Dirichlet boundary condition and distributed delay. We have shown that Hopf bifurcation occurs when the delay and the parameter k vary. The main techniques used here are usual, the analysis of the characteristic equation of the linearized problem, the Liapunov-Schmidt method and the Implicit Function Theorem.

Keywords. Periodic solutions, Hopf bifurcation, Competition reaction-diffusion, Distributed delay, Population dynamics.

AMS (MOS) subject classification: 35R10, 35B32, 35K55, 92D25.

1 Introduction

In many mathematical models of biological systems and population dynamic when the information of past states must be considered we include a negative delayed feedback control in the systems. We are concerned in the following reaction-diffusion system with delay which is incorporated to the control and Dirichlet boundary condition with competition between different species:

$$\begin{aligned} U_t(t,x) &= U_{xx}(t,x) + kU(t,x) + \frac{k}{\delta}\int_{-\tau}^{-\tau+\delta} g_1(U(t,x),U(t+s,x))\,ds + \\ &\quad \frac{k}{\delta}\int_{-\tau}^{-\tau+\delta} g_2(U(t,x),V(t+s,x))\,ds, \qquad (1.1)\\ V_t(t,x) &= V_{xx}(t,x) + kV(t,x) + \frac{k}{\delta}\int_{-\tau}^{-\tau+\delta} h_1(V(t,x),V(t+s,x))\,ds + \\ &\quad \frac{k}{\delta}\int_{-\tau}^{-\tau+\delta} h_2(V(t,x),U(t+s,x))\,ds,\;\; t>0,\; 0<x<\pi, \end{aligned}$$

$U(t,0) = U(t,\pi) = V(t,0) = V(t,\pi) = 0, \;\; t \geq 0,$

$(U(t,x),V(t,x)) = (\psi_1(t,x),\psi_2(t,x)), \;\; (t,x) \in [-\tau,0]\times[0,\pi],$

$(\psi_1,\psi_2) \in C([-\tau,0],H_0^1\times H_0^1)$, with k, τ e δ positive constants , $0<\delta\leq\tau$.

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