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L^p-SOLUTIONS OF RICCATI-TYPE DIFFERENTIAL EQUATIONS AND ASYMPTOTICS OF THIRD ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. We prove the existence of L^p-solutions for a second order nonlinear Riccati-type scalar differential equation. This is applied to a perturbed third order linear differential equation with L^p-perturbations for some $p \geq 1$. In this way, we know the asymptotic behavior of a solution y and its derivatives showing error bounds. Furthermore, we obtain an asymptotic expansion in terms of $L^{p/i}$ functions ($1 \leq i \leq [p]$, $[p]$ is the integer part of p), giving estimates for the error functions. The usefulness of these scalar results is shown studying the asymptotic behavior of a linear equation with unbounded coefficients. The asymptotic formulae obtained are simpler and more explicit than those obtained by 3×3 first order systems.

Keywords. Riccati type equation, Perturbed third order linear differential equation, Asymptotic formula, L^p-Perturbations, Error bounds.

AMS (MOS) subject classification: 34E10, 34E05.

1 Introduction

For the last decades, higher order differential equations have been intensively and extensively studied for representing a diversity of models in real world. The most of the applications arise from theoretical physics, population dynamics, biology, ecology, etc., see for example [7, 15, 16, 17, 18, 19, 22].

Particularly interesting are the differential equations with unbounded coefficients (see for example [1, 2, 3, 4, 7, 21, 22, 26])

$$y''' + qy' + ry = 0. \tag{1.1}$$

If either r or q is dominant, then equation (1.1) can be transformed in an almost constant third order differential equation [4, 18, 19, 21]

$$y''' + (a_2 + r_2(t))y'' + (a_1 + r_1(t))y' + (a_0 + r_0(t))y = 0. \tag{1.2}$$