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A Characterization of Compact Sets in R^n and its Application to a Geometric Inequality [0]

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Abstract. In this paper, we show that curvature conditions on a manifold control the topology of plane sections of the manifolds and then characterize compact n-manifold with C^2-boundary in terms of the topology of plane sections. We make use of the characterization to obtain a new lower bound for the integral of mean curvature for a certain kind of compact n-manifolds with C^2-boundary in R^n.

Keywords. height function, Morse theory, positive type or negative critical point, double, kinematic density, quasi-convex hull.

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1 Introduction

Let $M \subset R^{n+1}$ be a hypersurface of $(n+1)$-dimensional Euclidean spaces. In a neighborhood of any point p of M there is a unit normal vector field $\nu : M \to S^n \subset R^{n+1}$, unique up to sign, and hence a second fundamental form $II : T_pM \times T_pM \to R$ (which is defined up to sign). We also have the map $d\nu : T_pM \to T_pM$ with

$$\begin{aligned} II(X_p, Y_p) &= < -d\nu(X_p), Y_p > \\ &= - < \nabla_{X_p}\nu, Y_p > \\ &= < \nabla_{X_p}Y, \nu(p) > . \end{aligned}$$

But $< \nabla_{X_p}Y, \nu(p) >=< S(X_p, Y_p), \nu(p) >$, where $S(X_p, Y_p) = \perp (\nabla_{X_p}Y)$. Thus $-d\nu : T_pM \to T_pM$ is a symmetric linear transformation. We define principal curvatures at p to be the eigenvalues corresponding to the unit eigenvectors for $-d\nu$. It is obvious that all the principal curvatures at any point on the compact convex C^2-boundary are nonnegative.

The well-known characterization of compact convex set by G. Aumann

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