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INTERVAL OSCILLATION CRITERIA FOR FORCED SECOND-ORDER NONLINEAR DELAY DYNAMIC EQUATIONS WITH OSCILLATORY POTENTIAL

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Abstract. Differential equations with a forcing term have important applications in particle physics, especially as they arise from a certain radial solution of the Klein–Gordon equation which is the relativistic version of the Schrödinger equation and is used to describe spinless particles. In this paper, we will consider a forced nonlinear delay dynamic equation with an oscillatory potential function which as a special case includes differential and difference equations. We will use a unified approach on time scales and employ the Riccati technique to establish some criteria for oscillation. The results represent further improvements on those given earlier for differential and difference equations. Some examples are considered to illustrate the main results.
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1. Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his PhD thesis in 1988 in order to unify continuous and discrete analysis, see [**20**]. A time scale $\mathbb{T}$ is an arbitrary closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. Many other interesting time scales exist, and they give rise to many applications (see [**5**]).

This new theory of these so-called "dynamic equations" not only unifies the corresponding theories for the differential equations and difference equations cases, but it also extends these classical cases to cases "in between", e.g., to the so-called $q-$difference equations when $\mathbb{T}=q^{\mathbb{N}_0} := \{q^n : n \in \mathbb{N}_0 \text{ for } q > 1\}$, which has important applications in quantum theory (see [**23**]). Also the results can be applied on different types of time scales, which cover different types of equations when $\mathbb{T} = h\mathbb{N}$ for $h > 0$, $\mathbb{T} = \mathbb{N}^2$, $\mathbb{T} = \mathbb{T}_n = \{t_n : n \in \mathbb{N}\}$ where $\{t_n\}$ is the set of harmonic numbers, $\mathbb{T} = \mathbb{T}_2=\{\sqrt{n} : n \in \mathbb{N}_0\}$, and when $\mathbb{T} = \mathbb{T}_3=\{\sqrt[3]{n} : n \in \mathbb{N}_0\}$. We assume that the reader is familiar with the basic theory of the time scale calculus. If not we refer the reader to the books by Bohner and Peterson [**5**], [**6**] which summarize and organize much of the time scale calculus.

In recent years, there has been increasing interest in obtaining sufficient conditions for the oscillation/nonoscillation of solutions of different classes of dynamic equations on time scales. Following this trend, in this paper we are concerned with

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