*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **A:** *Mathematical Analysis* **17 (2010)** 487-499
 http://www.watam.org

# ON THE HEAT EQUATION DEFINED BY ITERATED LAPLACE BESSEL OPERATOR

Mehmet Zeki Sarikaya

Afyon Kocatepe University,
Department of Mathematics, Faculty of Science and Arts, Afyonkarahisar-TURKEY

email:sarikaya@aku.edu.tr, sarikayamz@gmail.com

**Abstract.** In this article, we study the solution of heat equation which is related to the spectrum and the kernel which is so called the generalized Laplace Bessel heat kernel. Moreover, such the generalized Laplace Bessel heat kernel has interesting properties and also related to the kernel of an extension of the Bessel heat equation.

**Keywords.** Heat kernel, Dirac-delta distribution, Laplace Bessel Operator, Spectrum.

**AMS (MOS) subject classification:** 46F10 and 31B10.

# 1 Introduction

The casual fundamental solution $C(x,t)$ is the particular solution of

$$\frac{\partial E}{\partial t} - a\Delta E = \delta(x)\delta(t)$$

which vanishes identically for $t < 0$. Thus $C(x,t)$ satisfies

$$\frac{\partial C}{\partial t} - a\Delta C = \delta(x)\delta(t),\ C \equiv 0 \text{ for } t < 0.$$

The causal fundamental solution $C(x,t)$ has a direct physical interpretation; it is the temperature distribution in a medium which is at zero temperature up to time $t = 0$, when a concentrated source is introduced at $x = 0$, this source instantaneously releasing a unit of heat. Although $C$ is defined for all $t$ and $x$, its calculation presents a problem only for $t > 0 (C = 0$ for $t < 0)$. This immediately suggests a slightly different point of view; for $t > 0$ no sources are present, so that $C$ satisfies the homogeneous equation and must reduce, at $t = 0+$, to a certain initial temperature. This initial temperature is the one to which the medium has been raised just after the introduction of an instantaneous concentrated source of unit strength. We now show that this initial temperature is $\delta(x)$.

It is known that the one-dimensional diffusion equation

$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2},$$