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EXISTENCE OF MULTIPLE HOMOCLINIC SOLUTIONS FOR A NONLINEAR ELLIPTIC BOUNDARY VALUE PROBLEM

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Abstract. Let $N \geq 2$ and $\mathcal{D} \subset \mathbb{R}^{N-1}$ be a bounded domain with a smooth boundary $\partial\mathcal{D}$. In this paper, we consider the existence of homoclinic solutions for nonlinear elliptic problem

$$\begin{cases} \Delta u + g(x,u) = 0 & \text{in } \mathbb{R} \times \mathcal{D} \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial(\mathbb{R} \times \mathcal{D}), \end{cases}$$

where $g \in C^1((\mathbb{R} \times \overline{\mathcal{D}}) \times \mathbb{R}, \mathbb{R})$ and ν denotes the outward pointing normal derivative to the boundary $\partial(\mathbb{R} \times \mathcal{D})$ of $\mathbb{R} \times \mathcal{D}$.

Keywords. Homoclinic solution, nonlinear elliptic problem, variational method, mountain pass critical point, parabolic flow.

1 Introduction

Let $N \geq 2$ and $\Omega \subset \mathbb{R}^N$ be a cylindrical domain, i.e., $\Omega = \mathbb{R} \times \mathcal{D}$, where $\mathcal{D} \subset \mathbb{R}^{N-1}$ is a bounded domain with a smooth boundary $\partial\mathcal{D}$. In the present paper, we consider the existence of homoclinic solutions of boundary value problem

$$\begin{cases} \Delta u + g(x,u) & = 0 \quad \text{in } \Omega \\ \frac{\partial u}{\partial \nu} & = 0 \quad \text{on } \partial\Omega \end{cases} \tag{P}$$

where $g \in C^1(\overline{\Omega} \times \mathbb{R}, \mathbb{R})$ and ν denotes the outward pointing normal derivative to $\partial(\mathbb{R} \times \mathcal{D})$. For $x \in \Omega$, we set $x = (x_1, y)$, where $x_1 \in \mathbb{R}$ and $y \in \mathcal{D}$. We impose the following conditions on g :

(g1) $g(x,z) \in C^1(\overline{\Omega} \times \mathbb{R}, \mathbb{R})$ and is $1-$periodic with respect to x_1;

(g2) $G(x,z) = \int_0^z g(x,\tau)d\tau$ is $1-$periodic with respect to z.

In [3], Rabinowitz considered the existence of spacially heteroclinic solutions of problem (P) under the assumption (g1), (g2) and an additional condition