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OSCILLATION CRITERIA FOR CERTAIN FORCED FIRST ORDER DIFFERENCE EQUATIONS WITH MIXED NONLINEARITIES[1]

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Abstract. New oscillation criteria are established for first order forced difference equations with mixed nonlinearities, which generalize and improve some recent results in literature.
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1 Introduction

In this paper, we consider the first order forced difference equation with mixed nonlinearities of type

$$\Delta x(n) - p(n)x(n+1) + \sum_{i=1}^{m} q_i(n)x^{\lambda_i}(n+1) = e(n), \tag{1}$$

where $\{p(n)\}, \{e(n)\}$ and $\{q_i(n)\}$ $(1 \le i \le m)$ are sequences of real numbers, and λ_i $(1 \le i \le m)$ are ratios of odd positive integers with $\lambda_1 > \cdots > \lambda_l > 1 > \lambda_{l+1} > \cdots > \lambda_m$.

By a solution of Equation (1) we mean a sequence $\{x(n)\}$ which is defined for $n \ge n_0 \in \mathbb{N}_0 = \{0, 1, 2, \cdots\}$ and satisfies Equation (1). Such a solution is said to be oscillatory if for every $n_1 \ge N_0$, there exists $n \ge n_1$ such that $x(n)x(n+1) \le 0$; otherwise, it is called nonoscillatory. Equation (1) is said to be oscillatory if all its solutions are oscillatory.

Numerous oscillation criteria for Equation (1) and various special cases have been obtained recently (see [1-5] and references cited therein). When $p(n) = e(n) = 0$, and there is only one nonlinear term in Equation (1), well known conditions for Equation (1) to be oscillatory are

$$\sum^{\infty} q_1(n) = \infty \text{ for } \lambda < 1, \text{ and } \sum^{\infty} q_1(n) = -\infty \text{ for } \lambda > 1. \tag{2}$$

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