

## PERMANENCE AND EXTINCTION IN A NONAUTONOMOUS T-PERIODIC COMPETITIVE LOTKA-VOLTERRA SYSTEM<sup>1</sup>

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**Abstract.** A nonautonomous T-periodic competitive Lotka-Volterra system is considered in this paper. For  $r \leq n$ , sufficient conditions on the coefficients are provided to guarantee that  $r$  of the species in the system are permanent while the remaining  $n - r$  are driven to extinction. It is shown that this result is an improvement of that in Ahmad and Montes de Oca [Appl. Math. Comput. **90**, (1998) 155-166].

**Keywords.** Lotka-Volterra system; Permanence; Extinction; Attractivity; Periodic solution.

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## 1 Introduction

We consider a nonautonomous Lotka-Volterra system of differential equations

$$\dot{x}_i(t) = x_i(t) \left[ b_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t) \right], \quad i = 1, \dots, n, \quad n \geq 2, \quad (1)$$

where  $x_i(t)$  represents the population size of the  $i$ th species at time  $t$ ,  $b_i(t)$ ,  $i = 1, \dots, n$ , and  $a_{ij}(t)$ ,  $i, j = 1, \dots, n$ , are continuous and T-periodic,  $a_{ii}(t) > 0$ ,  $i = 1, \dots, n$ , and  $a_{ij}(t) \geq 0$ ,  $i \neq j$ ,  $i, j = 1, \dots, n$ , for  $t \in (-\infty, +\infty)$ ,  $\dot{x}_i(t) = \frac{d}{dt}x_i(t)$ . Assume

$$\bar{b}_i = \frac{1}{T} \int_0^T b_i(s) ds > 0, \quad i = 1, \dots, n. \quad (2)$$

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