Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **17 (2010)** 397-407
 http://www.watam.org

DIFFERENT PHYSICAL STRUCTURES OF THE ASSETS FOR RATIONAL EXPECTATION MODELS

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Abstract. A generalized continuous time rational expectation model is reformulated as forward-backward stochastic differential equations (FBSDEs). Applying the Itô's formulae and the FBSDEs, we show that the assets in the model must satisfy a nonlinear differential equation. Three concrete examples are provided to state that the relation between the discount factor and the nonlinear coefficients in the rational expectation model determines the physical structures of the assets.

Keywords. Rational expectation model; Stochastic differential equations; Itô's formulae; Assets.

AMS (MOS) subject classification: 60H10, 91B02.

1 Introduction

For continuous time stochastic saddle point systems, Miller and Weller [1] investigated the rational expectation model

$$\begin{cases} dx_t = \alpha(x_t - x^*)dt + \beta(y_t - y^*)dt + \sigma dW_t, \\ y_t = E\Big[\int_t^\infty -\gamma(x_s - x^*)e^{-\delta(s-t)}ds|F_t\Big] + y^*, \quad \delta > 0, \end{cases} \tag{1}$$

where x_t is an economic fundamental and follows a diffusion process, y_t represents an asset whose price is a rational expectation forecast of properly discounted future fundamentals. The stars denote equilibrium states, which without loss of generality, are assumed to be equal to 0. The uncertainty is put into the model by a Wiener process W_t and the rational expectations of the fundamental are taken over the information set $F_t = \sigma(W_u, u \le t)$, the natural filtration generated by the Wiener process. The constant $\delta > 0$ is a discount factor. Miller and Weller [1] emphasized that a number of classic economic models are included in the model (1). The Blanchard's model relating stock market prices to the level of real activity in the economy (See [2]) is a typical example of model (1). Another model is the Krugman's system for the target zones (see [3]).

The original rational expectation model of Miller and Weller was considered to be linear. The nonlinear generalization of the Miller and Weller's model was investigated in [4] by Yannacopoulos, who discussed an s-dimensional

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