Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **17 (2010)** 347-355
 http://www.watam.org

PRESERVATION OF HOPF BIFURCATION UNDER THE EULER DISCRETIZATION OF DELAY DIFFERENTIAL SYSTEMS[1]

Yingxiang Xu[a,2] and Mingyou Huang[b]

[a]School of Mathematics & Statistics, KLAS of MOE,
Northeast Normal University, Changchun, P. R. China

[b]School of Mathematics,
Jilin University, Changchun, P. R. China

Abstract. In this paper, we consider the preservation of Hopf bifurcation under the Euler discretization of delay differential systems. We prove that there will be a Neimark-Sacker bifurcation, i.e., Hopf bifurcation for maps, occurs in the discretization at parameter value $\alpha(h) = \alpha^* + O(h)$ by the implicit function theorem under assumptions that the delay differential system undergoes a Hopf bifurcation at parameter value α^* and the stepsize h is sufficiently small.

Keywords. Delay differential system; Hopf bifurcation; Euler method; Preservation.

AMS (MOS) subject classification: 39A11; 65L20.

1 Introduction

In the numerical methods for delay differential systems, the works which discuss on the convergence and stability of numerical schemes appeared more and more commonly. And the introduction of all kinds of conceptions on stability, such as P-stability, GP-stability, D-stability etc. have deeply infected the researches on the numerical methods for delay differential systems. The works above mainly discuss whether the approximate solutions defined by the numerical schemes have similar stability properties or not when the solutions of continuous system have certain stability properties, and the problem of convergence. In other words, the numerical integrable methods for definite systems are discussed. However, the objects of study on bifurcation theory in dynamical systems are the bifurcation properties and the structures of the solutions of undefinite systems. Therefore, in the viewpoint of dynamical behavior, we need to investigate whether the discrete schemes preserve the bifurcation properties of the original dynamical systems or not, i.e., the research on the preservation of bifurcation behavior under the discretization of original systems, which is a brand-new research field of the numerical

[1]Supported by NSFC-10971022, 10701038, NCET-08-0755 and Science Foundation for Young Teachers of Northeast Normal University (No. 20090101).
[2]Corresponding author email:yxxu@nenu.edu.cn