Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **17 (2010)** 325-345
 http://www.watam.org

STABILITY AND HOPF BIFURCATION OF A MODEL FOR HIV INFECTION OF CD4$^+$ T CELLS

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Abstract. In the paper, we consider a mathematical model described human immunodeficiency virus (HIV) infection of CD4$^+$ T-cells. We discuss the existence and stability of the uninfected steady state and the infected steady state. Further, we consider the corresponding delay-differential equation model, and obtain conditions for the infected equilibrium to be asymptotically stable for all delay. Meanwhile, we study the effect of the time delay on the stability of the infected steady state and the existence of Hopf bifurcation, we also get the estimation of the length of delay to preserve stability. Numerical simulations are carried out to explain the mathematical conclusions.

Keywords. HIV infection; Hopf Bifurcation; steady state; stability

AMS subject classification: T34C05, 92D25.

1 Introduction

Mathematical modelling is of considerable importance in epidemiology because it may provide understanding of the underlying mechanisms which influence the spread of disease and may suggest control strategies. There has been many efforts in the mathematical modelling of the Acquired Immunodeficiency Syndrome (AIDS) epidemic and human immunodeficiency virus (HIV), see [1]-[11]. Mathematical models have played a significant role in the development of a better understanding of the disease and the various drug therapy strategies used against it ([12],[13]).

When the CD4$^+$ T cell count, which is normally around $1000mm^{-3}$, reaches $200mm^{-3}$ or below in an HIV-infected patient, then that person is classified as having AIDS. Because of the central role of CD4$^+$ T cells in immune regulation, their depletion has widespread deleterious on the functioning of the immune system as a whole and leads to the immunodeficiency that characterizes AIDS.