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A NEW PROOF OF A THEOREM OF DATKO AND PAZY

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Abstract. We give a very short proof for the well-known theorem of Datko and Pazy regarding the uniform exponential stability of abstract evolution families on half-line. Some other characterizations for the uniform exponential stability (and instability) of the evolution families are obtained.

Keywords. evolution families, uniform exponential stability, Datko-Pazy theorem.

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1 Introduction and Preliminaries

Let X be a real or complex Banach space and $B(X)$ the Banach algebra of all linear and bounded operators acting on X. We denote by $||\cdot||$ the norms of vectors and operators on X. We recall that a homomorphism $t \mapsto T(t)$, from $(I\!R_+, +)$ into $(B(X), \cdot)$, is called a (one-parameter) semigroup of linear and bounded operators on X. If in addition $T(\cdot)$ is strongly continuous (i.e. there exists $\lim\limits_{t\to 0_+} T(t)x = x$, for all $x \in X$) then $\{T(t)\}_{t\geq 0}$ is said to be a C_0-semigroup (for a general presentation of the theory of C_0-semigroups we refer the reader to [1]).

One of the most remarkable results concerning the (exponential) stability of a C_0-semigroup $\{T(t)\}_{t\geq 0}$ has been obtained in 1970 by R. Datko [2] and it says that all the trajectories $t \mapsto T(t)x$ have an exponential decay as $t \to \infty$ (i.e. $\{T(t)\}_{t\geq 0}$ is exponentially stable) if and only if, for each vector $x \in X$, the function $t \to \|T(t)x\|$ lies in $L^2(I\!R_+)$. Later, A. Pazy [7] shows that the result remains valid if we replace $L^2(I\!R_+)$ with $L^p(I\!R_+)$, where $p \in [1, \infty)$.

In 1972, R.Datko [3] extends the result above for abstract linear evolution families (see definition bellow) stating that an evolution family $\{U(t,s)\}_{t\geq s\geq 0}$ (with exponential growth) is uniformly exponentially stable (i.e. there are $N, \nu > 0$ such that $||U(t,t_0)|| \leq Ne^{-\nu(t-t_0)}$, for all $t \geq t_0 \geq 0$) if and only if there exists $p \in [1,\infty)$ such that $\sup\limits_{s\geq 0} \int\limits_s^\infty \|U(t,s)x\|^p dt < \infty$, for each $x \in X$. For proving this fact, Datko establishes firstly the equivalence between uniform exponential stability and uniform asymptotic stability (i.e.