Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **17 (2010)** 363-376
 http://www.watam.org

TRAJECTORIES OF A CHARGE IN A MAGNETIC FIELD ON RIEMANNIAN MANIFOLDS WITH BOUNDARY*

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Abstract. We prove an existence result for trajectories of classical particles accelerated by a potential and a magnetic field on a non–complete Riemannian manifold M. Both the potential and the magnetic field may be not bounded and have *critical growth.* We state a suitable convexity assumption involving the magnetic field in order to prove that the support of each trajectory is entirely contained in M.

Keywords. Riemannian manifolds with boundary, potential and magnetic field, asymptotic critical behavior, variational methods, minimum points.

AMS (MOS) subject classification: 58E10 53C20 70H03

1 Introduction

In this paper we deal with motions of classical particles on a Riemannian manifold (M, g_R) under the action of an electric force and a magnetic vector potential, both independent of time. More precisely, if $V : M \to \mathbb{R}$ is a smooth function, F is an exact two–form on M, $T > 0$ and $x_0, x_1 \in M$ are two fixed points of M, we look for curves $x : [0, T] \to M$ such that

$$\begin{cases} D_s\dot{x} + \frac{q}{m}\nabla V(x) = \frac{q}{m}\hat{F}(x)[\dot{x}] \\ x(0) = x_0, x(T) = x_1 \end{cases} \tag{1.1}$$

where $\frac{q}{m}$ is any fixed charge-to-mass ratio, D_s denotes the covariant derivative induced by the Levi–Civita connection, ∇ is the gradient with respect to g_R and $\hat{F} : TM \to TM$ is the linear map associated to F, that is

$$F(x)[u, v] = g_R(x)[\hat{F}(x)[u], v] \quad \forall x \in M, u, v \in T_xM.$$

*Work supported by M.I.U.R. (research project "Metodi variazionali e topologici ed equazioni differenziali non lineari")