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EXISTENCE RESULTS FOR SEMILINEAR FUNCTIONAL DIFFERENTIAL INCLUSIONS INVOLVING RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVE

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Abstract. In this paper, we shall establish sufficient conditions for the existence of mild solutions for some densely defined semilinear functional differential inclusions involving the Riemann-Liouville fractional derivative. The both cases of convex valued and nonconvex valued of the right hand side are considered. Our approach is based on the $\mathcal{C}_0$-semigroups theory combined with some suitable fixed point theorems. The topological structure of the solutions set is studied.

Keywords. Semilinear functional differential inclusions, fractional derivative, fractional integral, fixed point, selection, semigroups, mild solutions.

AMS (MOS) subject classification: 34G25, 26A33, 26A42.

1 Introduction

Functional differential equations arise in a variety of areas of biological, physical, and engineering applications, see, for example, the books of Hale [15], Hale and Verduyn Lunel [16], Kolmanovskii and Myshkis [24] and Wu [35], and the references therein. Recently, there has been a great deal of interest in optimal control systems described by differential equations in Banach spaces. These optimal control problems lead to differential inclusions and functional differential inclusions, where the present dynamics of systems are influenced by its past behavior; see for instance the monographs by Ahmed [2], Benchohra *et al* [3], Kisielewicz [23] and Smirnov [34]. Differential inclusions is a generalization of differential equations, therefore all problems considered for differential equations, that is, existence of solutions, continuation of solutions, dependence on initial conditions and parameters, are present in the theory of differential inclusions. Since a differential inclusion usually has many solutions starting at a given point, new issues appear, such as investi-