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EXISTENCE OF SOLUTION FOR ANTI-PERIODIC BOUNDARY VALUE PROBLEMS WITH IMPULSES

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Abstract. By using Schauder's fixed point theorem, we discuss the anti-periodic boundary value problem for a second order impulsive differential equations. Some sufficient conditions for existence of solution are obtained. Three examples are presented to illustrate our main results.

Keywords. Anti-periodic boundary value problem; Fixed point; Impulse; Existence; Differential equation.

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1 Introduction

In this paper, we consider the anti-periodic boundary value problems for second order impulsive differential equations of the form

$$\begin{cases} u'' + f(t, u(t)) = 0, \quad t \in J^* = J/\{t_1, t_2, ..., t_m\}, \\ \Delta u'(t_k) = J_k(u(t_k)), \ k = 1, 2, \cdots, m, \\ u(0) + u(T) = 0, \quad u'(0) + u'(T) = 0, \end{cases} \tag{1}$$

where $J = [0, T]$, $0 = t_0 < t_1 < t_2 < \cdots < t_m < t_{m+1} = T, f \in C(J \times R, R)$, $J_k \in C(R, R)$, $\Delta u'(t_k) = u'(t_k^+) - u'(t_k^-)$. Let $PC(J, R) = \{u : J \to R; u(t)$ is continuous everywhere except for t_k at which $u(t_k^+)$ and $u(t_k^-)$ exist and $u(t_k^-) = u(t_k), k = 1, 2, \cdots, m\}$; $PC^1(J, R) = \{u \in PC^1(J, R); u'(t)$ is continuous differentiable everywhere except for t_k at which $u'(t_k^+)$ and $u'(t_k^-)$ exist and $u'(t_k^-) = u'(t_k), k = 1, 2, \cdots, m\}$.

If a function $u \in PC^1(J, R) \cap C^2(J^*)$ satisfies equation (1), we call u a solution of (1).

In recent years, there has been a great deal of research on the problem of existence and uniqueness of solutions to anti-periodic boundary value problems. The existence of solutions for anti-periodic boundary value problems