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EXISTENCE RESULTS FOR AN IMPULSIVE SECOND ORDER DIFFERENTIAL EQUATION WITH STATE-DEPENDENT DELAY

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Abstract. By using the theory of cosine function of bounded linear operators, in this paper we study the existence of mild solutions for a class of second order impulsive abstract functional differential equations with state-dependent delay.

Keywords. Second order abstract Cauchy problem, impulsive differential equations, cosine function of operators, state dependent delay, unbounded delay.

AMS (MOS) subject classification: 34A37, 34K30, 47D09.

1 Introduction

In this paper, we study the existence of mild solutions for a second order impulsive abstract Cauchy problem with state-dependent delay described in the form

$$x''(t) = Ax(t) + f(t, x_{\rho(t,x_t)}), \quad t \in I = [0, a], \tag{1.1}$$
$$x_0 = \varphi \in \mathcal{B}, \quad x'(0) = \zeta \in X, \tag{1.2}$$
$$\Delta x(t_i) = I_i(x_{t_i}), \quad i = 1, 2, \dots, n, \tag{1.3}$$
$$\Delta x'(t_i) = J_i(x_{t_i}), \quad i = 1, 2, \dots, n, \tag{1.4}$$

where A is the infinitesimal generator of a strongly continuous cosine function of bounded linear operator $(C(t))_{t\in\mathbb{R}}$ defined on a Banach space X, the function $x_s : (-\infty, 0] \to X$, $x_s(\theta) = x(s+\theta)$, belongs to some abstract phase space $\mathcal{B}$ described axiomatically, $0 < t_1 < \cdots < t_n < a$ are prefixed numbers, $f : I \times \mathcal{B} \to X$, $\rho : I \times \mathcal{B} \to (-\infty, a]$, $I_i(\cdot) : \mathcal{B} \to X$, $J_i(\cdot) : \mathcal{B} \to X$ are appropriate functions and the symbol $\Delta\xi(t)$ represents the jump of the function $\xi(\cdot)$ at t, which is defined by $\Delta\xi(t) = \xi(t^+) - \xi(t^-)$.