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BIFURCATION ANALYSIS ON THE FITZHUGH-NAGUMO NEURAL MODEL

Amirhossein Hajihosseini[1] and Gholam Reza Rokni Lamooki[1,2]

[1]Center of Excellence in Biomathematics, School of Mathematics, Statistics and Computer Science, University of Tehran, Tehran, Iran, *hajihosseini@khayam.ut.ac.ir*

[2]College of Science, School of Mathematics, Statistics and Computer Science, University of Tehran, Tehran, Iran, *rokni@khayam.ut.ac.ir*

Abstract. This paper studies the FitzHugh-Nagumo neural model from the bifurcation view point using the center manifold reduction. A three-d view of the fold bifurcation manifold will be presented. By the help of the normal form theorem, we observe that when the system has a unique equilibrium, it undergoes a generic Hopf bifurcation.

Keywords. FitzHugh-Nagumo neural model; Fold bifurcation; Hopf bifurcation; Normal form theorem; Center manifold theorem.

AMS (MOS) subject classification: Primary 92B20, 34C23, Secondary 37C27, 37G05.

1 Introduction

Electric signaling or firing by neurons has been long well-known. By using differential equations, Hodgkin and Huxley [7] were the first to introduce a mathematical model for the nerve axon of the squid. They proposed a four dimensional dynamical system with one equation for the membrane voltage and three others for inward sodium and outward potassium currents through their corresponding channels in the membrane; see [5, 12] for more details on the model. In this paper, we work on a neural impulse model which is a reduction of the Hodgkin-Huxley model (the HH model) to a two dimensional system. This reduced mathematical model is known as the FitzHugh-Nagumo model, or simply the FHN model, and is studied separately by FitzHugh [2] and Nagumo et al. [13].

The FHN model has been studied by mathematicians under different parameter conditions. The global asymptotic stability of an equilibrium has been considered in [1, 4]. The stability of the traveling wave solutions of the FHN of reaction-diffusion equations have been investigated in [3, 9, 18]. Recently, Rocsoreanu et al. [14] have studied a specific form of the FHN system. They have carried out a full investigation on this type of FHN system and studied in detail the conditions for the parameters under which some important codimension one and two bifurcations take place. In another work, Rocsoreanu et