*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **A:** *Mathematical Analysis* **17 (2010)** 215-232
 http://www.watam.org

# EXISTENCE AND REGULARITY OF WEAK SOLUTIONS FOR THE BIHARMONIC EQUATION WITH COMPLETE SECOND ORDER DERIVATIVE

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**Abstract.** In this paper, we discuss the biharmonic equations with complete second order derivative terms. We prove that the equation admits a unique weak solution under some conditions. Via some techniques and difference quotients method, we also show the interior regularity and the boundary regularity of the weak solution. These techniques include cut-off function, coordinate transposition, flatten out the boundary, finitely covered theorem, general Sobolev inequalities, interpolation inequality, etc..

**Keywords.** Biharmonic equation, Existence and Uniqueness, Difference quotients, Interior regularity, Boundary regularity.

**AMS (MOS) subject classification:** 31A30, 35B65, 35J30.

# 1 Introduction

Biharmonic equation arises in many physical problems, such as the bending of clamped thin elastic isotropic plates, equilibrium of an elastic body under conditions of plane strain or plane stress, and creeping flow of a viscous incompressible fluid. At the same time, many problems may also be translated into biharmonic problem. For example, the Stokes flow (2D) can be described in terms of the biharmonic problem. Therefore, many authors have studied biharmonic problems; see, for instance, [1,2,3,4] and references therein. On the other hand, in the field of equations, we can obtain some solutions of differential equations by mean of operator theory or calculus of variations. However, this may lead to a question; whether the solutions thus obtained are sufficiently smooth to be solutions in a classical sense. Fortunately, this question may be solved by regularity theory (see[5]). There are so many references of biharmonic equation, therefore we can only list a few of them associated with our question.