EQUILIBRIUM PROBLEMS AND MOUFAFI’S VISCOSITY APPROXIMATION METHODS IN HILBERT SPACES

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Abstract. We establish an iterative scheme by means of Mann’s method and Moudafi’s method to find a common element of the set of solutions of an equilibrium problem and the set of fixed points of a nonexpansive mapping in a Hilbert space. We prove a convergence theorem of our iteration under the weaker assumption as were the case in Takahashi and Takahashi’s recent results. The new iteration considered in the paper is applied to find a common element of the set of fixed points of a nonexpansive mapping and the set of solutions of a variational inequality problem for continuous monotone mappings. Consequently, the corresponding results for \(\alpha\)-inverse-strongly monotone mappings, \(r\)-strongly monotone mappings and relaxed \((\gamma, r)\)-cocoercive mappings are obtained respectively. We also propose a slightly modified Mann-type iteration to obtain a strong convergence theorem for continuous pseudocontractive mappings.

Keywords. viscosity approximations method, equilibrium problem, variational inequality problem, nonexpansive mapping, monotone mapping, pseudocontractive mapping

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1 Introduction

Let \(H\) be a real Hilbert space and \(C\) be a nonempty closed convex subset of \(H\). Let \(F\) be a bifunction of \(C \times C\) into \(\mathbb{R}\), where \(\mathbb{R}\) is the set of real numbers. The equilibrium problem for \(F: C \times C \to \mathbb{R}\) is to find \(x \in C\) such that

\[
F(x, y) \geq 0 \quad \text{for all } y \in C. \tag{1}
\]

The set of solutions of (1) is denoted by \(\text{EP}(F)\). Given a mapping \(T: C \to H\), let \(F(x, y) = \langle Tx, y - x \rangle\) for all \(x, y \in C\). Then, \(z \in \text{EP}(F)\) if and only if \(\langle Tz, y - z \rangle \geq 0\) for all \(y \in C\), i.e., \(z\) is a solution of the variational inequality. Numerous problems in physics, optimization, and economics reduce to find a solution of (1). Some methods have been proposed to solve the equilibrium problem (see [1, 5, 13]). In 2005, Combettes and Hirstoaga [4] introduced an iterative scheme of finding the best approximation to the initial data when \(\text{EP}(F)\) is nonempty and they also proved a strong convergence theorem.