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EQUILIBRIUM PROBLEMS AND MOUDAFI'S VISCOSITY APPROXIMATION METHODS IN HILBERT SPACES

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Abstract. We establish an iterative scheme by means of Mann's method and Moudafi's method to find a common element of the set of solutions of an equilibrium problem and the set of fixed points of a nonexpansive mapping in a Hilbert space. We prove a convergence theorem of our iteration under the weaker assumption as were the case in Takahashi and Takahashi's recent results. The new iteration considered in the paper is applied to find a common element of the set of fixed points of a nonexpansive mapping and the set of solutions of a variational inequality problem for continuous monotone mappings. Consequently, the corresponding results for α-inverse-strongly monotone mappings, r-strongly monotone mappings and relaxed (γ, r)-cocoercive mappings are obtained respectively. We also propose a slightly modified Mann-type iteration to obtain a strong convergence theorem for continuous pseudocontractive mappings.
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1 Introduction

Let H be a real Hilbert space and C be a nonempty closed convex subset of H. Let F be a bifunction of $C \times C$ into $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. The equilibrium problem for $F : C \times C \to \mathbb{R}$ is to find $x \in C$ such that

$$F(x, y) \geq 0 \quad \text{for all } y \in C. \tag{1}$$

The set of solutions of (1) is denoted by $\mathrm{EP}(F)$. Given a mapping $T : C \to H$, let $F(x, y) = \langle Tx, y - x \rangle$ for all $x, y \in C$. Then, $z \in \mathrm{EP}(F)$ if and only if $\langle Tz, y - z \rangle \geq 0$ for all $y \in C$, i.e., z is a solution of the variational inequality. Numerous problems in physics, optimization, and economics reduce to find a solution of (1). Some methods have been proposed to solve the equilibrium problem (see [1, 5, 13]). In 2005, Combettes and Hirstoaga [4] introduced an iterative scheme of finding the best approximation to the initial data when $\mathrm{EP}(F)$ is nonempty and they also proved a strong convergence theorem.