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STABILITY CRITERIA FOR FIRST ORDER DELAY DIFFERENTIAL EQUATIONS WITH M COMMENSURATE DELAYS

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Abstract. In this paper we study the asymptotic stability of the zero solution of first order linear delay differential equations of the form

$$y'(t) + \sum_{j=0}^{m} a_j y(t - j\tau) = 0$$

where $a_j,\ j = 0, \ldots, m$, are certain constants and $\tau > 0$ is constant. A new necessary condition for asymptotic stability is obtained, and a robust algorithmic test for asymptotic stability is also obtained. In two special cases for two delays, considerable simplifications and refinements of the test are also provided. In proving our results, we make use of Pontryagin's theory for quasi-polynomials and of Chebyshev polynomials.

Keywords. asymptotic stability, stability criteria, algorithmic stability test, commensurate delays, characteristic functions, stability regions, Chebyshev polynomials.

AMS (MOS) subject classification: 45E99, 34D99.

1 Introduction

The aim of this paper is to study the asymptotic stability of the zero solution of the delay differential equation

$$y'(t) + \sum_{j=0}^{m} a_j y(t - j\tau) = 0 \tag{1}$$

where $\tau > 0$, a_j, $j = 0, \ldots, m$ are constants. Equation (1) has many applications in Biology [15] and Control [11,21]. In a previous paper [4], we considered equation (1) with one delay and complex coefficients. Although there has been some study of differential equations with more than one delay (see [8,13,20] and references therein), there has been no comprehensive study for several commensurate delays. Some work on stability with respect to small