

## MEAN-VARIANCE PORTFOLIO OPTIMIZATION UNDER STOCHASTIC INCOME AND UNCERTAIN EXIT TIME

Zhongfei Li<sup>1</sup> and Shuxiang Xie<sup>2</sup>

<sup>1</sup>Department of Risk Management and Insurance, Lingnan College, Sun Yat-sen University, Guangzhou 510275, P.R. China,  
E-mail: [lnslzf@mail.sysu.edu.cn](mailto:lnslzf@mail.sysu.edu.cn)

<sup>2</sup>Department of Probability and Statistics, School of Mathematics and Computational Science, Sun Yat-Sen University, Guangzhou 510275, P.R. China,  
E-mail: [xieshuxiang@gmail.com](mailto:xieshuxiang@gmail.com)

**Abstract.** In the present work we give a generalization of the continuous-time mean-variance portfolio selection problem of Zhou and Li (2000) on four points of view: (i) the market is incomplete; (ii) the dividend rate and the gain process are considered; (iii) the optimal income-financing strategy is discussed; (iv) the uncertain exit time is allowable. In our model, both the risky assets' price dynamics and the stochastic income are governed by geometric Brownian motions, and the uncertain exit time is a continuous random variable that is independent of the Brownian motion. The objective of the investor is to maximize the expected terminal wealth while minimizing the variance of the terminal wealth. By using the embedding technique, we first establish an auxiliary certain exit time linear quadratic control problem which is equivalent to the uncertain exit time optimization problem, then we explicitly derive the optimal income-financing strategy by using the general stochastic linear quadratic control theory. Finally the efficient frontiers of the terminal wealth is obtained in closed-forms. The main results of Zhou and Li (2000) turn out to be a special case of our results.

**Keywords.** Portfolio Optimization; Stochastic Income; Uncertain Exit Time; Mean-Variance Model; Linear-Quadratic Control

**AMS (MOS) subject classification:** 93E20; 91B28

## 1 Introduction

Recently, portfolio choice with stochastic income has been treated in a number of articles. For example, [11] studies a consumption and portfolio selection problem with uninsurable labor income risk in a complete market; [5] investigates an optimal portfolio selection model for investors with negative exponential utility and faced with imperfectly hedgeable stochastic income risk; [10] investigates the optimal portfolio choice among inflation-linked bonds, nominal bonds, and stocks for investors endowed with an indexed stream of income. More discussion on income can be found in [2, 3, 14, 19] and the references therein.