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POSITIVE SOLUTIONS FOR NONLINEAR SECOND-ORDER DIFFERENCE EQUATIONS WITH IMPULSE

Aydin Huseynov

Institute of Mathematics and Mechanics, Azerbaijan National Academy of Sciences
AZ1141 Baku, Azerbaijan

Abstract. In this study, we investigate nonlinear second order difference equations subject to linear impulse conditions and separated linear boundary conditions. Sign properties of an associated Green's function are exploited to get existence results for positive solutions of the nonlinear boundary value problem with impulse. Upper and lower bounds for positive solutions are also given.

Keywords. Impulsive difference equations, Green's function, Positive solution, Fixed point theorems.

AMS (MOS) subject classification: 39A10

1 Introduction

The significance of investigation of positive (nonnegative) solutions is due to the fact that in analysing nonlinear phenomena many mathematical models give rise to problems for which only nonnegative solutions make sense. For proof of existence of positive solutions to various nonlinear boundary valuue problems the theory of operators (nonlinear, in general) acting in Banach spaces with a cone and leaving this cone invariant has proved to be enough fruitful; see the monographs [1, 8, 12] and papers [2, 3, 4, 7]. Differential equations with impulses are a basic tool to study processes that are subjected to abrupt changes in their state. There has been a significant development in the last two decades [5, 6, 13, 15]. Impulsive difference equations have started to be considered quite recently [9, 14, 16−18].

Let $\mathbb{Z}$ denote the set of all integers. For any $l, m \in \mathbb{Z}$ with $l \leq m$, $[l, m]$ will denote the *discrete interval* being the set $\{l, l+1, \ldots, m\}$. Semi-infinite intervals of the form $(-\infty, l]$ and $[l, \infty)$ will denote the discrete sets $\{\ldots, l-2, l-1, l\}$ and $\{l, l+1, l+2, \ldots\}$, respectively. Throughout the paper all intervals will be discrete intervals.

In this paper, we investigate the existence of positive solutions for the following boundary value problem with impulse (BVPI):

$$-\Delta[p(n-1)\Delta y(n-1)]+q(n)y(n) = f(n, y(n)), \quad n \in [a, c-1]\cup[c+2, b], \tag{1}$$

$$y(c-1) = d_1 y(c+1), \quad y^{[\Delta]}(c-1) = d_2 y^{[\Delta]}(c+1), \tag{2}$$