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EXISTENCE AND UNIQUENESS RESULTS FOR NONLINEAR FIRST-ORDER IMPLICIT IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS WITH MONOTONE CONDITIONS [1]

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Abstract. In this paper, by using a monotone iterative technique in the presence of lower and upper solutions, we discuss the existence of solutions for a new class of nonlinear first-order implicit impulsive integro-differential equations in Banach spaces. Under wide monotone conditions and the noncompactness measure conditions, we also obtain the existence of extremal solutions and a unique solution between lower and upper solutions. Our results improve and extend some relevant results in abstract differential equations.
Keywords. Nonlinear first-order implicit impulsive integro-differential equation, monotone iterative technique, monotone condition and noncompactness measure condition, lower and upper solution, existence and uniqueness.
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1 Introduction

Let $\mathbb{B}$ be a Banach space, $J = [t_0, t_0 + a] \subset R = (-\infty, +\infty)$ is a compact interval, $f : J \times \mathbb{B} \times \mathbb{B} \times \mathbb{B} \to \mathbb{B}$ is continuous, $\lambda_i \geq 0$ $(i = 1, 2)$ is a constant, and $Tu(t) = \int_{t_0}^{t} h(t,s)u(s)ds$, $h(t,s) \in C(D, \mathbb{R}^+)$, $\mathbb{R}^+ = [0, +\infty)$, $D = \{(t,s)|s,t \in J, t \geq s\}$, $I_k \in C[\mathbb{B}, \mathbb{B}]$ for $k = 1, 2, \cdots, m$. For $u_0 \in \mathbb{B}$, we consider the following new class of nonlinear first-order implicit impulsive integro-differential equation problem: find $u : J \to \mathbb{B}$ such that

$$\begin{cases} u'(t) = f(t, u(t), \lambda_1 Tu(t), \lambda_2 u'(t)), \ t \neq t_k, \\ \triangle u|_{t=t_k} = I_k(u(t_k)), \ k = 1, 2, \cdots, m, \\ u(t_0) = u_0, \end{cases} \tag{1.1}$$

where $t_0 < t_1 < \cdots < t_m < t_0 + a < +\infty$, $\triangle u|_{t=t_k}$ denotes the jump of $u(t)$ at $t = t_k$, i.e., $\triangle u|_{t=t_k} = u(t_k^+) - u(t_k^-)$, $u(t_k^-)$ and $u(t_k^+)$ represent the left and right limits of $u(t)$ at $t = t_k$, respectively.

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