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POSITIVE SOLUTIONS FOR SINGULAR BOUNDARY VALUE PROBLEMS OF COUPLED SYSTEMS OF DIFFERENTIAL EQUATIONS

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Abstract. This paper deals with the existence and multiplicity of positive solutions for singular boundary value problems of nonlinear ordinary differential systems

$$\begin{cases} (-1)^m u^{(2m)} = \lambda a(t) f(t, u(t), v(t)), & a.e.\ t \in [0,1], \\ (-1)^n v^{(2n)} = \mu b(t) g(t, u(t), v(t)), & a.e.\ t \in [0,1], \\ u^{(2i)}(0) = u^{(2i)}(1) = 0, & 0 \le i \le m-1, \\ v^{(2j)}(0) = v^{(2j)}(1) = 0, & 0 \le j \le n-1, \end{cases}$$

where $\lambda > 0, \mu > 0, m, n \in \mathbf{N}$, f and g are Caratheodory functions. Under suitable conditions we derive two explicit intervals, and such that λ and μ in the two intervals respectively. Furthermore, the existence and multiplicity of positive solutions for λ and μ in appropriate intervals is also discussed. Our approach is based on Krasnosel'skii fixed point theorem on compression and expansion of cones.

Keywords. Positive solution, Nonlinear ordinary differential system, Singular boundary value problem.

AMS (MOS) subject classification: 34B15,34B16,34B18.

1 Introduction

The purpose of this paper is to establish the existence of single and multiple positive solutions to a class of singular boundary value problems for systems of higher order nonlinear differential equations:

$$\begin{cases} (-1)^m u^{(2m)} = \lambda a(t) f(t, u(t), v(t)), & a.e.\ t \in [0,1], \\ (-1)^n v^{(2n)} = \mu b(t) g(t, u(t), v(t)), & a.e.\ t \in [0,1], \\ u^{(2i)}(0) = u^{(2i)}(1) = 0, & 0 \le i \le m-1, \\ v^{(2j)}(0) = v^{(2j)}(1) = 0, & 0 \le j \le n-1, \end{cases} \tag{1.1}$$

where $m, n \in \mathbf{N}, \lambda > 0, \mu > 0$ are positive parameters. $a(t), b(t) \in L^1[[0,1],[0,+\infty)]$, f and g are essential bounded functions(see Definition 2.1 of Section 2).

Here a positive solution (u^*, v^*) of (1.1) will mean a solution (u^*, v^*) of (1.1) satisfying $u^*(t) \ge 0, v^*(t) \ge 0, t \in (0,1)$.