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A PROCEDURE TO SOLVE A SINGULAR STOCHASTIC OPTIMAL CONTROL PROBLEM

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Abstract. In this work we deal with a singular stochastic optimal control problem. We present a theoretical iterative method which converges to the analytical solution and we also present a discretization procedure to obtain an approximated solution. We establish the convergence of the discrete solution to the value function and give an example of application with the numerical results.

Keywords. singular controls, Hamilton-Jacobi-Bellman equation, viscosity solution, iterative constructive method, numerical solution

AMS (MOS) subject classification: 49L20, 49L25, 49M25, 93E20

1 Introduction

In this paper we consider a stochastic control problem where the state is governed by the following stochastic differential equation

$$x_t = x + \int_s^t b(\theta, x_\theta, u_\theta)d\theta + \int_s^t \sigma(\theta, x_\theta, u_\theta)dB_\theta + \int_s^t g(\theta)dv_\theta. \qquad (1)$$

We denote with $(\Xi, F, F_t, \mathbf{P})$ the probability framework, where F_t is an increasing set of σ-algebras defined on Ξ, $F = \bigcup_t F_t$ and $\mathbf{P}$ is a probability measure defined on the elements of F. b, σ, g are deterministic functions and $(B_t, t \geq 0)$ is a d-dimensional Brownian motion, x is the initial position of the system at time s, and the controls are $u \in \mathcal{U}$ and $v \in \mathcal{V}$, where

$$\mathcal{U} = \{u : [0,T] \to U \subset \mathbb{R}^c : u \text{ non anticipative w.r.t. } F_t\},$$

$$\mathcal{V} = \left\{v : [0,T] \to \mathbb{R}^k_+ : \forall p = 1, ..., k,\ v_p \text{ non decr., non anticip. w.r.t.} F_t\right\}.$$

The expected cost for each pair of controls has the form

$$J(s, x, u, v) = E\left\{\int_s^T f(t, x_t, u_t)dt + \int_s^T c(t)dv_t\right\},$$